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Pump Noise Transfer in Parametric Amplifiers

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Abstract—**Pump noise transfer in paramps is related to the sensitivity of amplifier gain to AM and FM pump modulation. Measurements performed on an *S*-band paramp with an 18.4-GHz pump are described. The significance of the results is made explicit by using them to calculate the increase in amplifier noise temperature associated with an IMPATT oscillator pump.**

I. INTRODUCTION

The noise figure of parametric amplifiers has been observed [1]–[3] to increase as a result of pump noise, when a strong signal is present in the passband. Of the various oscillators available for pump sources, the IMPATT oscillator has the greatest noise content and, thus, is more prone to produce this effect. This note presents data on pump noise transfer and an estimate of the effect produced by IMPATT oscillators. The basic premise in this work is that pump noise causes the amplifier gain to vary, and that sinusoidal pump modulation can be used to obtain quantitative results about this phenomenon.

II. ANALYSIS

Modulation of the pump amplitude and frequency, E_p and f_p, produces variations in the amplitude and phase, $G^{1/2}$ and ϕ, of the voltage reflection coefficient, and this, in turn, modulates the output of any signal E_s applied to the amplifier. For harmonic pump modulation at frequency f_m, the variation of the reflection coefficient and output voltage can be formally expressed as follows:

$$\delta G^{1/2} = \left(\frac{\partial G^{1/2}}{\partial E_p}\right)\delta E_p + \left(\frac{\partial G^{1/2}}{\partial f_p}\right)\delta f_p = \left(\frac{\partial G^{1/2}}{\partial E_p}\right)2\Delta E_p \cos\,(2\pi f_m t + \theta_E) + \left(\frac{\partial G^{1/2}}{\partial f_p}\right)2\Delta f_p \cos\,(2\pi f_m t + \theta_f)$$

$$\delta\phi = \left(\frac{\partial \phi}{\partial E_p}\right)\delta E_p + \left(\frac{\partial \phi}{\partial f_p}\right)\delta f_p = \left(\frac{\partial \phi}{\partial E_p}\right)2\Delta E_p \cos\,(2\pi f_m t + \theta_E) + \left(\frac{\partial \phi}{\partial f_p}\right)2\Delta f_p \cos\,(2\pi f_m t + \theta_f)$$

$$\delta E_{so} = (\delta G^{1/2} + jG^{1/2}\delta\phi)\exp\,(j\phi)E_{si}$$

where ΔE_p and Δf_p are, respectively, the voltage and frequency amplitudes of the AM and FM sidebands, and θ_E and θ_f are arbitrary phase constants.

The amplitude and frequency modulation of the pump are now considered separately. For pure AM on the pump ($\Delta f_p = 0$), the power $P_{so}(\text{AM})$ in each of the output signal sidebands is given by

$$P_{so}(\text{AM}) = \left\{\left(\frac{\partial G^{1/2}}{\partial E_p}\right)^2 + G\left(\frac{\partial \phi}{\partial E_p}\right)^2\right\}(\Delta E_p)^2 P_{si} = K(\text{AM})\left(\frac{P_p(\text{AM})}{P_p}\right)(GP_{si}) \quad (1)$$

where P_{si} is the input signal power, $[P_p(\text{AM})/P_p]$ is the pump sideband to carrier ratio, and $K(\text{AM})$ is the AM sensitivity coefficient defined by

$$K(\text{AM}) \equiv \left(\frac{P_p}{G}\frac{\partial G}{\partial P_p}\right)^2 + \left(2P_p\frac{\partial \phi}{\partial P_p}\right)^2.$$

For pure FM on the pump ($\Delta E_p = 0$), the result is

$$P_{so}(\text{FM}) = \left\{\left(\frac{\partial G^{1/2}}{\partial f_p}\right)^2 + G\left(\frac{\partial \phi}{\partial f_p}\right)^2\right\}(\Delta f_p)^2 P_{si} = K(\text{FM})\left(\frac{f_m}{f_p}\right)^2\left(\frac{P_p(\text{FM})}{P_p}\right)(GP_{si}) \quad (2)$$

where the sideband to carrier ratio is related to the frequency deviation by

$$\frac{P_p(\text{FM})}{P_p} = \left(\frac{\Delta f_p}{f_m}\right)^2$$

and the FM sensitivity coefficient is defined by

$$K(\text{FM}) = \left(\frac{f_p}{2G}\frac{\partial G}{\partial f_p}\right)^2 + \left(f_p\frac{\partial \phi}{\partial f_p}\right)^2.$$

Equations (1) and (2) provide the analytic framework for the measurements described in the following sections. The general defining relations for $K(\text{AM})$ and $K(\text{FM})$ have been applied to a specific model for a parametric amplifier. The salient result of this work is that the sensitivity parameters are proportional to the amplifier gain. A detailed description of this work will be presented in a forthcoming paper on the theory of pump noise transfer in parametric amplifiers.

III. MEASUREMENTS

Measurements were performed on an AIL 381088 parametric amplifier which had a double-tuned signal circuit centered at 2.24 GHz and an 18.4-GHz Gunn oscillator pump source. The gain response curve for this amplifier is shown in Fig. 1. The measurements took the following form: the pump was sinusoidally modulated, a strong input signal was applied to the amplifier, and the modulation sidebands on the amplifier output signal were monitored. The parameters that were independently varied included the input signal frequency, input signal power, pump modulation frequency, and pump modulation ratios.

The measured data had the general behavior described in (1) and (2)—that is, the output signal sidebands increased linearly with the pump modulation ratio and the input signal power. In addition, the AM results were independent of the modulating frequency, while the FM data showed an average increment of 6 dB/octave, as expected. Equations (1) and (2) were used to convert the raw data for the various experimental conditions into values of the sensitivity co-

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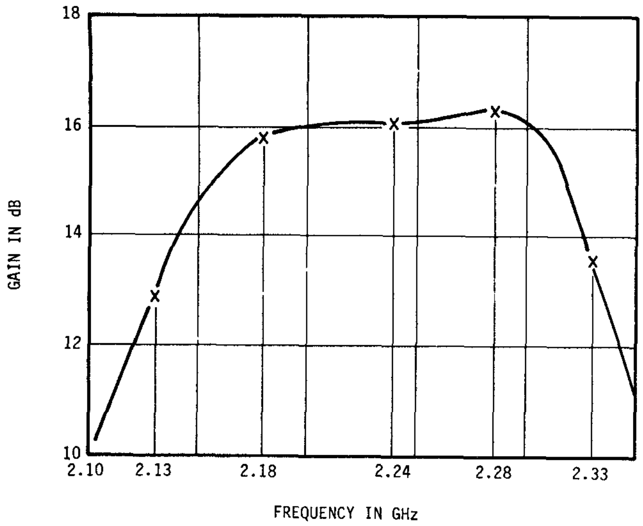

Fig. 1. Paramp gain curve and measurement frequencies.

efficients. The values of $K(\text{AM})$ obtained for

$$(f_m; P_p(\text{AM})/P_p) = (1\text{ MHz}; -59\text{ dB}),\ (4.5\text{ MHz}; -59\text{ dB}),$$

and

$$(4.5\text{ MHz}; -64\text{ dB})$$

were averaged and are presented in Table I. The mean deviation was generally less than 5 percent. The sensitivity coefficient, which is expected to depend on the gain, shows a marked variation with signal frequency and a tendency to peak at 2.28 GHz. This tendency is probably related to the specific operating conditions and double-tuning network of the amplifier tested. The results are quite consistent for the different input signal levels, except again, at the high frequency end, where distortions occur for an input of −35 dBm. The values of $K(\text{FM})$ averaged for

$$(f_m; P_p(\text{FM})/P_p) = (4.5\text{ MHz}; -59\text{ dB}),\ (4.5\text{ MHz}; -64\text{ dB}),$$

and

$$(10\text{ MHz}; -59\text{ dB})$$

are presented in Table II. The output signal sidebands produced by FM on the pump were much weaker than those produced by AM, and in those cases indicated by an X, the signal was below the noise level of the receiver. The results for AM and FM were supplemented by measurements performed using single-sideband pump modulation. When the single sideband was 6 dB larger than a corresponding AM sideband, both forms of modulation produced the same results.

The results obtained for sinusoidal pump modulation have been used to estimate the effect of noise from real sources. Levine *et al.* [4], quote measured values of AM and FM noise in 100-Hz bands for GaAs IMPATT oscillators operating at 10 GHz. Scaling their results to a bandwidth B, located f_m away from an 18.4-GHz operating frequency, we obtain the following for the noise-to-carrier ratios:

$$\frac{N_p(\text{AM})}{P_p} \approx 10^{-15.7}B \qquad (3)$$

$$\frac{N_p(\text{FM})}{P_p} = \left(\frac{\Delta f_{\text{rms}}}{2f_m}\right)^2 \approx \frac{B}{f_m^2}. \qquad (4)$$

The FM noise ratio was scaled in proportion to the square of the operating frequency, and both ratios were scaled linearly with bandwidth. The resultant output noise power in the signal circuit was calculated from these noise ratios by use of (1) and (2). For this purpose, average midband values, $K(\text{AM}) = 21$ dB and $K(\text{FM}) = 62$ dB, were used. The increase in amplifier noise temperature associated with the transferred noise power is then given by

TABLE I
RESULTS FOR AMPLITUDE MODULATED PUMP

P_S (dBm)	Signal Frequency (GHz)					
	2.10	2.13	2.18	2.24	2.28	2.33
-45	11	13	20	20	22	20
-40	10	14	20	21	24	20
-35	10	13	19	23	34	24

Note: Average values of $K(\text{AM})$ are tabulated in decibels for an amplifier with a midband gain of 16 dB.

TABLE II
RESULTS FOR FREQUENCY MODULATED PUMP

P_S (dBM)	Signal Frequency (GHz)					
	2.10	2.13	2.18	2.24	2.28	2.33
-40	X	X	63	62	64	X
-35	X	X	61	63	72	64

Note: Average values of $K(\text{FM})$ are tabulated in decibels for an amplifier with a midband gain of 16 dB. X means data reading in the noise.

$$\Delta T(\text{AM}) = \frac{N_s(\text{AM})}{GkB} = 10^{6.2}P_{si} \qquad (5)$$

$$\Delta T(\text{FM}) = \frac{N_s(\text{FM})}{GkB} = 10^{5.4}P_{si} \qquad (6)$$

where k is Boltzmann's constant and the units of ΔT and P_{si} are degrees kelvin and milliwatts, respectively. These relations are in fair agreement with the measured results of Clunie *et al.* [1].

The paramp used in the measurements had a noise temperature of 100 K. Thus, according to (5) and (6), the midband noise temperature would double for an input signal of −42 dBm, and the principal cause would be transferred AM noise. This conclusion is predicated on an IMPATT oscillator with the noise content stated in (3) and (4) and an amplifier with 16 dB of gain.

IV. APPARATUS

The experimental arrangement used is shown in Fig. 2. Pump power from the Gunn oscillator was coupled into a diode amplitude modulator and then coupled back into the pump line leading to the amplifier. The form of the resultant modulation in the pump line could be continuously shifted between pure AM and pure FM by adjusting the line stretcher in the tapped line—that is, by changing the phase of the modulation relative to the carrier. The amplitude of the modulation was observed on the Polarad spectrum analyzer, and the form of the modulation was determined by use of the crystal detector. As the line stretcher was varied, the modulation displayed on the analyzer remained fixed, while the output of the detector was maximum for pure AM and zero for pure FM. Single-sideband modulation was obtained by coupling power from a signal generator into the tapped line and monitoring it on the Polarad analyzer. The amplitude of the modulation and carrier were independently adjusted by separate attenuators. Directional couplers, isolators, and tuners were used to achieve the necessary control of power flow.

With an input signal applied to the amplifier from an S-band generator, the amplified output signal was observed on an AIL 707 spectrum analyzer. The sensitivity of the spectrum analyzer and the allowable input signal levels were limiting factors in these measurements. The gain of the amplifier became distorted if the input exceeded −35 dBm and the spectrum analyzer was unable to detect the FM sidebands if the input signal fell below −40 dBm. Another factor limiting the precision of the FM measurements was the degree to which pure FM was obtained. The data recorded were generally ±1 dB and the smallest detectable signal was about −95 dBm.

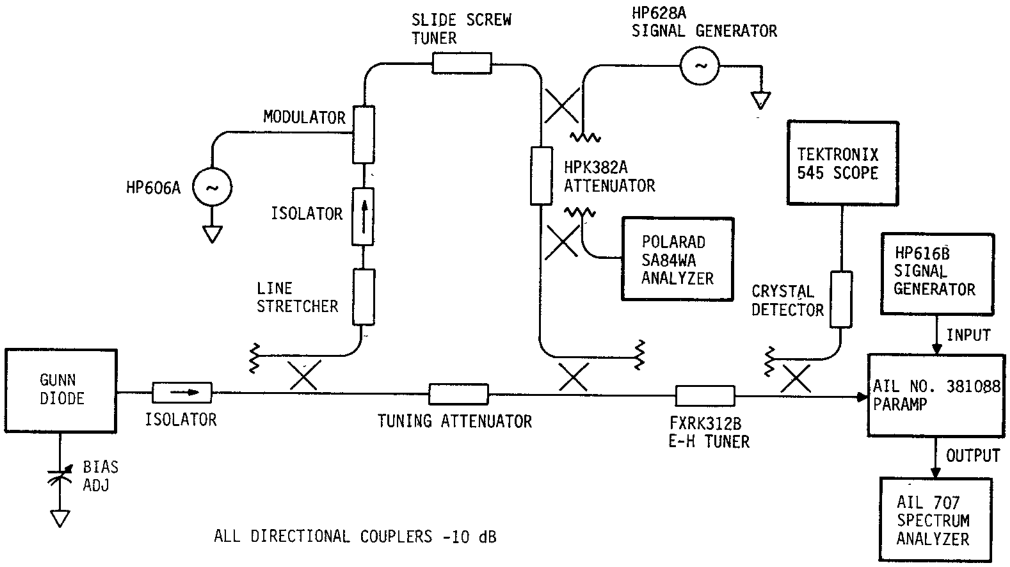

Fig. 2. Experimental arrangement.

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A Technique for Computing Dispersion Characteristics of Shielded Microstrip Lines

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***Abstract*—The boundary value problem associated with the shielded microstrip-line structure is formulated in terms of a rigorous hybrid-mode representation. The resulting equations are subsequently transformed, via the application of Galerkin's method in the spectral domain, to yield a characteristic equation for the dispersion properties of shielded microstrip lines. Among the advantages of the method are its simplicity and rapid convergence. Numerical results are included for several different structural parameters. These are compared with other available data and with some experimental results.**

I. INTRODUCTION

With the increasing use of microstrip-line circuits at higher frequencies, a number of workers have studied the dispersion properties of microstrip lines [1]. Mittra and Itoh [2] used the singular integral equation approach for the shielded microstrip line, while Krage and Haddad [3] employed the Fourier-series expansion of the fields in the shielded microstrip line followed by a point-matching technique.

Itoh and Mittra have recently demonstrated the application of a novel technique, called the spectral-domain analysis, to the problem of determining the dispersion characteristics of open microstrip lines [4]. In this short paper the technique just mentioned is extended to apply to the shielded microstrip-line problem. Some of the unique features of the method, which is based on the application of Galerkin's method in the finite Fourier transform domain, are as follows: a) the spectral method is numerically simpler and more efficient than the conventional space-domain techniques. This is due primarily to the fact that in the present method solutions are extracted from algebraic equations rather than from coupled integral equations typically appearing in the conventional space-domain approaches; b) the use of transforms allows one to convert convolutions into algebraic products, thus avoiding the necessity of numerical evaluation of complicated integrals, a process which is often extremely time consuming; c) typically, eigenvalues for the propagation constants are obtained from a determinantal equation, and the behavior of the field distribution of the eigenmodes are not readily discernible [2]. However, in the present method the physical nature of the field corresponding to each mode is directly incorporated in the process of solution via the appropriate choice of basis functions.

II. FORMULATION OF THE PROBLEM

Fig. 1 shows the cross section of the shielded microstrip line. The structure is assumed to be uniform and infinite in the z direction. The infinitely thin strip and the shield case are perfect conductors. It is also assumed that the substrate material is lossless and its relative permittivity and permeability are ϵ_r and μ_r, respectively.

The hybrid-field components in the microstrip line can be expressed in terms of a superposition of the TE and TM fields, which are in turn derivable from the scalar potentials $\psi^{(e)}$ and $\psi^{(h)}$. For instance

$$E_{zi} = j\frac{k_i^2 - \beta^2}{\beta}\psi_i^{(e)}(x,y)\exp(-j\beta z) \tag{1a}$$

$$H_{zi} = j\frac{k_i^2 - \beta^2}{\beta}\psi_i^{(h)}(x,y)\exp(-j\beta z) \tag{1b}$$

$$k_1 = \omega(\epsilon_r\mu_r\epsilon_0\mu_0)^{1/2}, \quad k_2 = \omega(\epsilon_0\mu_0)^{1/2} = k_0 \tag{2}$$

where β is the unknown propagation constant and ω is the frequency. The superscripts (e) and (h) are associated with the TM- and TE-type fields, respectively. The subscripts $i = 1,2$ serve to designate region 1 (substrate) or 2 (air). Other field components can be easily derived from Maxwell's equations.

From this point on, we proceed essentially in a manner similar to [4]. However, the bounded nature of the geometry of the shielded structure requires the use of the finite Fourier transform instead of the conventional Fourier transform over an infinite range. The

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